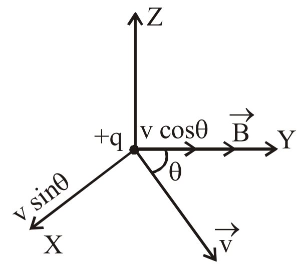
MOVING CHARGES AND MAGNETISM

GENERAL KEY CONCEPT

1. **Force on a moving charge:–** A moving charge is a source of magnetic field.



Let a positive charge q is moving in a uniform magnetic field $\vec{B}$ with velocity $\vec{v}$.'

$F \propto q \Rightarrow F \propto v \sin\theta \Rightarrow F \propto B$

$\therefore$ $F \propto qBv \sin\theta \Rightarrow F = kq\,Bv \sin\theta$ [k = constant]

k = 1 in S.I. system.

$\therefore$ $F = qBv \sin\theta$ and $\vec{F} = q(\vec{v} \times \vec{B})$

2. **Magnetic field strength** $(\vec{B})$**:**

In the equation $F = qBv \sin\theta$, if q = 1, v = 1,

$\sin\theta = 1$ i.e. $\theta = 90°$ then F = B.

$\therefore$ Magnetic field strength is defined as the force experienced by a unit charge moving with unit velocity perpendicular to the direction of magnetic field.

Special Cases:

(1) It $\theta = 0°$ or $180°$, $\sin\theta = 0$

$\therefore$ F = 0

A charged particle moving parallel to the magnetic field, will not experience any force.

(2) If v = 0, F = 0

A charged particle at rest in a magnetic field will not experience any force.

(3) If $\theta = 90°$, $\sin\theta = 1$ then the force is maximum

$F_{max.} = qvB$

A charged particle moving perpendicular to magnetic field will experience maximum force.

3. **S.I. unit of magnetic field intensity.** It is called tesla (T).

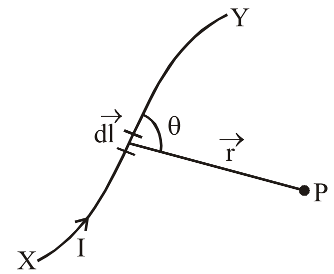
$$B = \frac{F}{qv\sin\theta}$$

If q = 1C, v = 1m/s, $\theta = 90°$ i.e. $\sin\theta = 1$ and F = 1N

Then B = 1T.

The strength of magnetic field at a point is said to be 1T if a charge of 1C while moving at right angle to a magnetic field, with a velocity of 1 m/s experiences a force of 1N at that point.

4. **Biot-Savart's law:–** The strength of magnetic field or magnetic flux density at a point P (dB) due to current element dl depends on,



(i) $dB \propto I$

(ii) $dB \propto dl$

(iii) $dB \propto \sin\theta$

(iv) $dB \propto \frac{1}{r^2}$,

Combining, $dB \propto \frac{Idl\sin\theta}{r^2} \Rightarrow dB = k\frac{Idl\sin\theta}{r^2}$ [k = Proportionality constant]

In S.I. units, $k = \frac{\mu_0}{4\pi}$ where μ_0 is called permeability of free space.

$\mu_0 = 4\pi \times 10^{-7}\ TA^{-1}m$

$\therefore\ dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$ and $d\vec{B} = \frac{\mu_0}{4\pi}I\frac{(\vec{dl}\times\vec{r})}{r^3}$

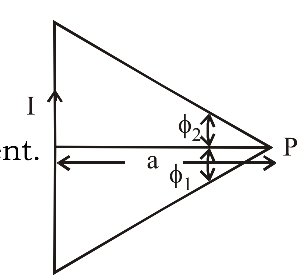
$d\vec{B}$ is perpendicular to the plane containing $d\vec{\ell}$ and $\vec{r}$ and is directed inwards.

5. **Applications of Biot-Savart's law:–**

(a) Magnetic field (B) at the Centre of a Circular Coil Carrying Current.

$B = \frac{\mu_0 nI}{2r}$

where n is the number of turns of the coil. I is the current in the coil and r is the radius of the coil.



(b) Magnetic field due to a straight conductor carrying current.

$B = \frac{\mu_0 I}{4\pi a}(\sin\phi_2 + \sin\phi_1)$

where a is the perpendicular distance of the conductor from the point where field is to the measured.

ϕ_1 and ϕ_2 are the angles made by the two ends of the conductor with the point.

(c) For an infinitely long conductor, $\phi_1 = \phi_2 = \pi/2$

$\therefore\ B = \frac{\mu_0}{4\pi}.\frac{2I}{a}$

(d) Magnetic field at a point on the axis of a Circular Coil Carrying Current.

when point P lies far away from the centre of the coil.

$B\ \frac{0}{4}.\frac{2M}{x^3}$

where M = nIA = magnetic dipole moment of the coil.

x is the distance of the point where the field is to the measured, n is the number of turns, I is the current and A is the area of the coil.

6. Ampere's circuital law:–

The line integral of magnetic field $\vec{B}$ around any closed path in vacuum is μ_0 times the total current through the closed path. i.e. $\oint \vec{B}.d\vec{l} = \mu_0 I$

7. Application of Ampere's circuital law:–

(a) Magnetic field due to a current carrying solenoid, $B = \mu_0 nI$

n is the number of turns per unit length of the solenoid.

At the edge of a short solenoid, $B = \frac{\mu_0 nI}{2}$

(b) Magnetic field due to a toroid or endless solenoid

$B = \mu_0 nI$

8. Motion of a charged particle in uniform electric field:–

The path of a charged particle in an electric field is a parabola.

Equation of the parabola is $x^2 = \frac{2mv^2}{qE} y$

where x is the width of the electric field.

y is the displacement of the particle from its straight path.

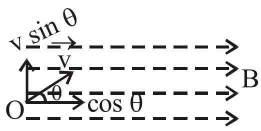
v is the speed of the charged particle.

q is the charge of the particle

E is the electric field intensity.

m is the mass of the particle.

9. Motion of the charged particle in a magnetic field. The path of a charged particle moving in a uniform magnetic field $(\vec{B})$ with a velocity $\vec{v}$ making an angle θ with $\vec{B}$ is a helix.



The component of velocity v cosθ will not provide a force to the charged particle, so under this velocity the particle with move forward with a constant velocity along the direction of $\vec{B}$. The other component v sinθ will produce the force F = q Bv sinθ, which will supply the necessary centripetal force to the charged particle in moving along a circular path of radius r.

$\because$ Centripetal force = $\frac{m(v\sin\theta)^2}{r}$ = B qv sinθ

$\therefore$ v sinθ = $\frac{Bqr}{m}$

Angular velocity of rotation = w = $\frac{v\sin\theta}{r} = \frac{Bq}{m}$

Frequency of rotation = $\nu = \frac{\omega}{2\pi} = \frac{Bq}{2\pi m}$

Time period of revolution = T = $\frac{1}{\nu} = \frac{2\pi m}{Bq}$

10. **Cyclotron:** It is a device used to accelerate and hence energies the positively charged particle. This is done by placing the particle in an oscillating electric field and a perpendicular magnetic field. The particle moves in a circular path.

$\therefore$ Centripetal force = magnetic Lorentz force

$$\Rightarrow \frac{mv^2}{r} = Bqv \Rightarrow \frac{mv}{Bq} = r \leftarrow \text{radius of the circular path}$$

Time to travel a semicircular path = $\frac{\pi r}{v} = \frac{\pi m}{Bq}$ = constant.

If v_0 be the maximum velocity of the particle and r_0 be the maximum radius of its path then

$$\frac{mv_0^2}{r_0} = Bqv_0 \Rightarrow v_0 = \frac{Bqr_0}{m}$$

Max. K.E. of the particle = $\frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\frac{Bqr_0}{m}\right)^2 \Rightarrow (K.E.)_{max.} = \frac{B^2q^2r_0^2}{2m}$

Time period of the oscillating electric field $\Rightarrow T = \frac{2\pi m}{Bq}$.

Time period is independent of the speed and radius.

Cyclotron frequency = $\nu = \frac{1}{T} = \frac{Bq}{2\pi m}$

Cyclotron angular frequency = $\omega_0 = 2\pi\nu = \frac{Bq}{m}$

11. Force on a current carrying conductor placed in a magnetic field:

$\vec{F} = I\left|\vec{\ell} \times \vec{B}\right|$ or $F = I\ell B\sin\theta$

where I is the current through the conductor

B is the magnetic field intensity.

l is the length of the conductor.

θ is the angle between the direction of current and magnetic field.

(i) When θ = 0° or 180°, $\sin\theta$ = 0 $\Rightarrow$ F = 0

$\therefore$ When a conductor is placed along the magnetic field, no force will act on the conductor.

(ii) When θ = 90°, $\sin\theta$ = 1, F is maximum.

$F_{max} = I\ell B$

when the conductor is placed perpendicular to the magnetic field, it will experience maximum force.

12. Force between two parallel conductors carrying current:–

(a) When the current is in same direction the two conductors will attract each other with a force

$F = \frac{\mu_0}{4\pi}.\frac{2I_1I_2}{r}$ per unit length of the conductor

(b) When the current is in opposite direction the two conductors will repel each other with the same force.

(c) S.I. unit of current is 1 ampere. (A).

1A is the current which on flowing through each of the two parallel uniform linear conductor placed in free space at a distance of 1 m from each other produces a force of 2×10^{-7} N/m along their lengths.

13. Torque on a current carrying coil placed in a magnetic field:–

$\vec{\tau} = \vec{M} \times \vec{B}$ $\Rightarrow$ τ = MB sinα = nIBA sinα where M is the magnetic dipole moment of the coil.

M = nIA

where n is the number of turns of the coil.

I is the current through the coil.

B is the magnetic field intensity.

A is the area of the coil.

α is the angle between the magnetic field $(\vec{B})$ and the perpendicular to the plane of the coil.

Special Cases:

(i) If the coil is placed parallel to magnetic field θ = 0°, cosθ = 1 then torque is maximum.

$$\tau_{max.} = nIBA$$

(ii) If the coil is placed perpendicular to magnetic field, θ = 90°, cosθ = 0

$\therefore$ τ = 0

14. Moving coil galvanometer:– This is based on the principle that when a current carrying coil is placed in a magnetic field it experiences a torque. There is a restoring torque due to the phosphor bronze strip which brings back the coil to its normal position.

In equilibrium, Deflecting torque = Restoring torque

nIBA = kθ [k = restoring torque/unit twist of the phosphor bronze strip]

$I = \frac{k}{nBA}\theta = G\theta$ where $G = \frac{k}{nBA}$ = Galvanometer constant

$\therefore$ $I \propto \theta$

Current sensitivity of the galvanometer is the deflection produced when unit current is passed through the galvanometer.

$$I_s = \frac{\theta}{I} = \frac{nBA}{k}$$

Voltage sensitivity is defined as the deflection produced when unit potential difference is applied across the galvanometer.

$V_s = \frac{\theta}{V} = \frac{\theta}{IR} = \frac{nBA}{kR}$ [R = Resistance of the galvanometer]

15. Condition for the maximum sensitivity of the galvanometer:-

The galvanometer is said to be sensitive if a small current produces a large deflection.

$\because \quad \theta = \frac{nBA}{k} I$

$\therefore \quad \theta$ will be large if (i) n is large, (ii) B is large (iii) A is large and (iv) k is small.

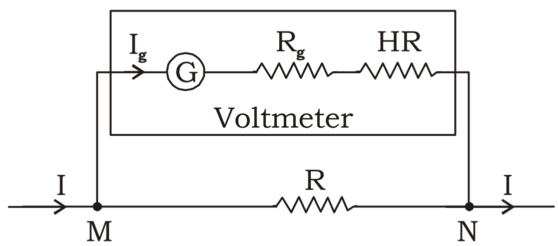
16. Conversion of galvanometer into voltmeter and ammeter

(a) A galvanometer is converted to voltmeter by putting a high resistance in series with it.

Total resistance of voltmeter = R_g + R where R_g is the galvonometer resistance. R is the resistance added in series.

Current through the galvanometer = $I_g = \frac{V}{Rg + R}$

where V is the potential difference across the voltmeter.

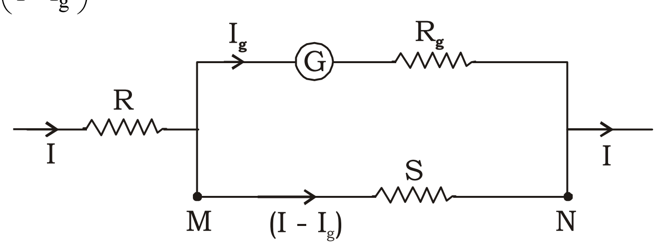


$\therefore \quad R = \frac{V}{I_g} - G$

Range of the voltmeter: 0 – V volt.

(b) A galvanometer is converted into an ammeter by connecting a low resistance in parallel with it (shunt)

Shunt = $S = \left(\frac{I_g}{I - I_g}\right) R_g$ where R_g is the galvanometere resistance.



I is the total current through the ammeter.

I_g is the current through the ammeter. Effective resistance of the ammeter

$R = \frac{R_g}{R_g + S}$

The range of the ammeter is 0 – I A. An ideal ammeter has zero resistance.